Equilibrium exchange rate model for Uzbekistan: VECM estimation for trade elasticity

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TECHNICAL NOTE UZBEKISTAN

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Content

1. Introduction

The task is to develop, estimate, and evaluate a time series model of the equilibrium exchange rate for Uzbekistan. The goal is to estimate the trade balance elasticity with respect to the real (effective) exchange rate.

The first step is to replicate a model constructed for Ukraine in 2009 (Weber et al., 2009). That model included four variables: the country's real GDP, the real effective exchange rate, the trade balance, and the real world GDP.

All calculations are performed using EViews.

2. Data

First, I collected the following macroeconomic variables. The table below defines these variables and lists their sources. All time series are quarterly.

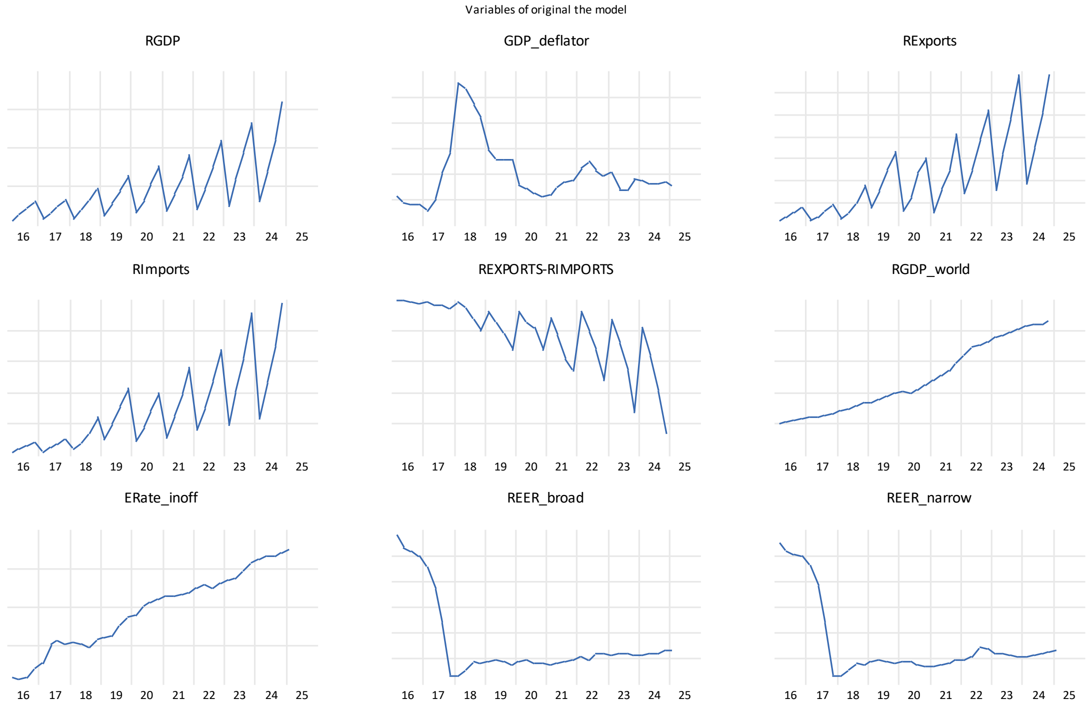
Table 1: Macroeconomic variables

Variable	Definition	Source
Gross domestic product (by expenditures method, at constant prices, quarterly)	previous year prices, national currency, billion soums	National Statistics committee of the Republic of Uzbekistan
Gross domestic product deflator index (quarterly)	as a percentage of the corresponding period of the previous year	National Statistics committee of the Republic of Uzbekistan
Export of goods and services (at constant prices, quarterly)	previous year prices, national currency, billion soums	National Statistics committee of the Republic of Uzbekistan
Imports of goods and services (at constant prices, quarterly)	previous year prices, national currency, billion soums	National Statistics committee of the Republic of Uzbekistan
Real world GDP	Gross domestic product (GDP), Price deflator, Seasonally adjusted (SA), Index	International Monetary Fund
Exchange rate	soums per US dollar	colleagues from National Bank of Uzbekistan
Real effective exchange rate	Uzbekistan, FX Indices, Bruegel, Real Effective Exchange Rate - Broad Index (REER)	Macrobond
Real effective exchange rate	Uzbekistan, FX Indices, Bruegel, Real Effective Exchange Rate - Narrow Index (REER)	Macrobond

The main data source is the National Statistics Committee of Uzbekistan.

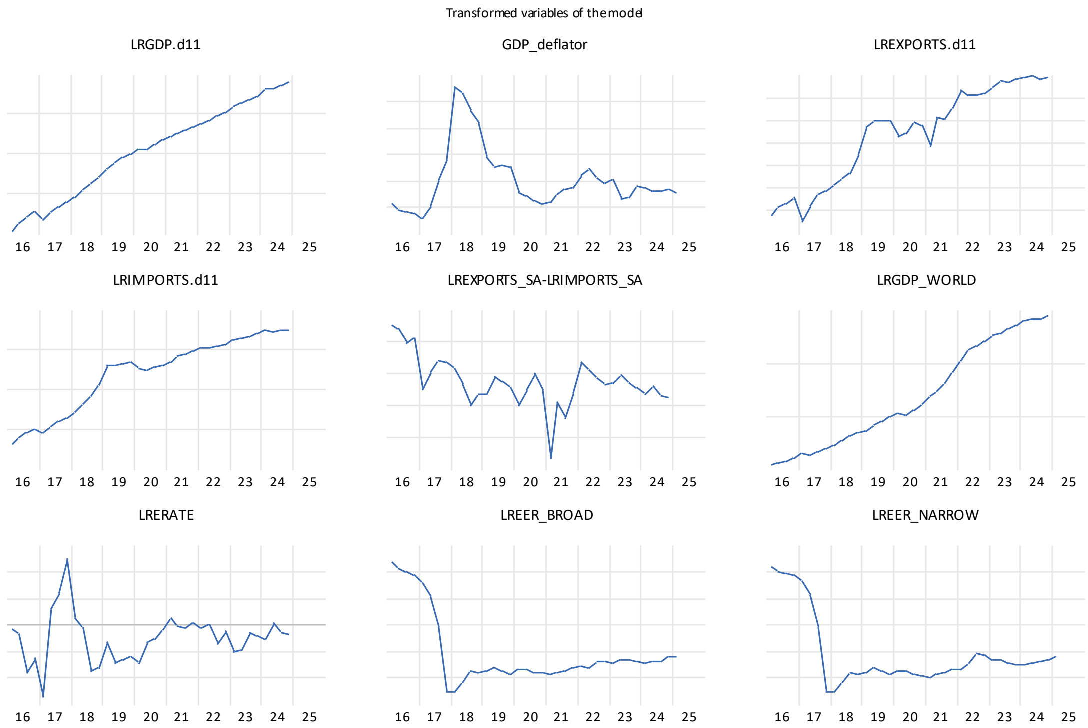
Before I can start analysis, I undertake a visual inspection of data to identify possible data problems. The original time series are shown in Figure 1.

Figure 1: The original macroeconomic variables



It is evident that real GDP, real exports, and real imports have a strong seasonal component. Therefore, they must be seasonally adjusted. This was done using the Census X13 method. Moreover, real effective exchange rate has a structural break in 2017. It is likely to be caused by the structural break in the official exchange rate that happened in September 2017. Therefore, one idea is to use the unofficial exchange rate that does not have such break and compute a proxy for the real exchange rate as a ratio between the index of the nominal exchange rate and the GDP deflator. The resulting transformed time series are shown in Figure 2.

Figure 2: Transformed macroeconomic variables



It can be seen that the seasonal adjustment effectively removed the seasonal components. In addition, the dynamics of the calculated real exchange rate differ before 2018 compared to the dynamics of both real effective exchange rates (broad and narrow). Therefore, I decided to use the sample from 2018Q3 to 2024Q4. Moreover, in what follows I am going to use two alternative proxies for the exchange rate: the calculated real exchange rate and the broad real effective exchange rate.

3. Testing

Reflecting the general comments, in the next sections we will suggest more detailed refinements and amendments to individual articles and paragraphs.

3.1. Cointegration

First, I test for cointegration. In the case that there is no cointegration, there is no need in estimating a vector error correction model (VECM). I conduct the Johansen cointegration test using two alternative sets of data: 1) real GDP, real trade balance, real world GDP, and real exchange rate and 2) real GDP, real trade balance, real world GDP, and real effective exchange rate.

Table 1: Cointegration test for data including real exchange rate

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.** Critical Value
None *	0.648102	51.89694	47.85613	0.0199
At most 1	0.473314	26.83101	29.79707	0.1058
At most 2	0.288530	11.44337	15.49471	0.1857
At most 3	0.127493	3.273239	3.841465	0.0704

Trace test indicates 1 cointegrating equation(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Max-eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.** Critical Value
None	0.648102	25.06593	27.58434	0.1016
At most 1	0.473314	15.38764	21.13162	0.2627
At most 2	0.288530	8.170130	14.26460	0.3617
At most 3	0.127493	3.273239	3.841465	0.0704

Max-eigenvalue test indicates no cointegration at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

The unrestricted cointegration rank test based on trace conducted for the data set including the real exchange rate found at least one cointegration relation. At the same time, the maximum-eigenvalue test found no cointegration at the 5% significance level.

Table 2: Cointegration test for data including real effective exchange rate

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.** Critical Value
None *	0.694391	58.39422	47.85613	0.0038
At most 1 *	0.532583	29.94349	29.79707	0.0481
At most 2	0.355727	11.69068	15.49471	0.1724
At most 3	0.046370	1.139503	3.841465	0.2858

Trace test indicates 2 cointegrating equation(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Max-eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.** Critical Value
None *	0.694391	28.45073	27.58434	0.0387
At most 1	0.532583	18.25281	21.13162	0.1207
At most 2	0.355727	10.55118	14.26460	0.1782
At most 3	0.046370	1.139503	3.841465	0.2858

Max-eigenvalue test indicates 1 cointegrating equation(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

In the case of the data set with real effective exchange rate, the Johansen tests (trace and maximum eigenvalue) found between one and two cointegrating equations. Hence, we can conclude that there is some evidence of the existence of at least one cointegrating relation in the data.

3.2. Lag order selection

Next, we test for the optimal number of lags using all information criteria available in the EViews. These tests are carried out for the vector autoregressions (VAR) estimated for the two alternative sets of data: 1) real GDP, real trade balance, real world GDP, and real exchange rate and 2) real GDP, real trade balance, real world GDP, and real effective exchange rate.

Table 3: Optimal lag order for data including real exchange rate

Lag	LogL	LR	FPE	AIC	SC	HQ
0	191.7865	NA	4.53e-13	-17.07150	-16.87313	-17.02477
1	289.0364	150.2952	2.90e-16	-24.45785	-23.46600	-24.22420
2	297.4136	9.900359	6.88e-16	-23.76487	-21.97953	-23.34430
3	312.5866	12.41428	1.22e-15	-23.68969	-21.11087	-23.08220
4	372.4019	27.18878*	8.62e-17*	-27.67290*	-24.30059*	-26.87849*

Note: LR: sequential modified LR test statistic (each test at 5% level); FPE: Final prediction error; AIC: Akaike information criterion; SC: Schwarz information criterion; HQ: Hannan-Quinn information criterion.

The information criteria computed for the first data set indicate an optimal order of four lags.

Table 4: Optimal lag order for data including real effective exchange rate

Lag	LogL	LR	FPE	AIC	SC	HQ
0	192.4010	NA	4.28e-13	-17.12737	-16.92900	-17.08064
1	281.8293	138.2073*	5.58e-16*	-23.80266	-22.81081*	-23.56901
2	294.9228	15.47419	8.63e-16	-23.53844	-21.75310	-23.11787
3	306.4316	9.416228	2.14e-15	-23.13014	-20.55131	-22.52265
4	345.8920	17.93656	9.60e-16	-25.26291*	-21.89059	-24.46849*

For the first data set, three out of five criteria suggest an optimal order of one lag, while the remaining two indicate an optimal order of four lags.

4. Estimation

Our final step is to estimate VECMs for both data sets and to display cumulative impulse responses. For the data set including real exchange rate I computed VECM with three lags because the use of four lags turns out to be impossible leading to near singularity.

Table 5: VECM for data including real effective exchange rate

Vector Error Correction Estimates

Sample (adjusted): 2019Q3 2024Q4

Included observations: 22 after adjustments

Standard errors in ( ) & t-statistics in [ ]

Lags interval (in first differences): 1 to 3

Endogenous variables: TBALANCE LRGDP_SA LRERATE LRGDP_WORLD

Deterministic assumptions: Case 3 (Johansen-Hendry-Juselius): Cointegrating relationship includes a constant. Short-run dynamics include a constant.

Cointegrating Eq:	CointEq1
TBALANCE(-1)	1.000000
LRGDP_SA(-1)	-1.092544
	(0.14379)
	[-7.59809]
LRERATE(-1)	-5.867438
	(0.52131)
	[-11.2552]
LRGDP_WORLD(-1)	5.122466
	(0.55840)
	[9.17352]

C	-10.52754			
Error Correction:	D(TBALANCE)	D(LRGDP_SA)	D(LRERATE)	D(LRGDP_WORLD)
COINTEQ1	-0.029218	0.114250	0.007709	-0.035585
	(0.04623)	(0.07306)	(0.05245)	(0.02003)
	[-0.63206]	[1.56381]	[0.14697]	[-1.77673]
D(TBALANCE(-1))	-0.420845	-0.225192	0.036596	0.244584
	(0.32264)	(0.50991)	(0.36610)	(0.13978)
	[-1.30440]	[-0.44164]	[0.09996]	[1.74972]
D(TBALANCE(-2))	-0.267353	-0.320419	0.088417	0.198643
	(0.39044)	(0.61707)	(0.44304)	(0.16916)
	[-0.68474]	[-0.51926]	[0.19957]	[1.17428]
D(TBALANCE(-3))	0.015292	-0.062603	-0.113801	0.051133
	(0.36203)	(0.57216)	(0.41079)	(0.15685)
	[0.04224]	[-0.10942]	[-0.27703]	[0.32600]
D(LRGDP_SA(-1))	-0.058994	-0.563955	0.106710	0.033105
	(0.27378)	(0.43269)	(0.31066)	(0.11862)
	[-0.21548]	[-1.30337]	[0.34350]	[0.27909]
D(LRGDP_SA(-2))	0.007609	0.004678	-0.190209	0.007780
	(0.17973)	(0.28405)	(0.20394)	(0.07787)
	[0.04233]	[0.01647]	[-0.93266]	[0.09991]
D(LRGDP_SA(-3))	0.047133	-0.133580	-0.230484	0.027797

	(0.19734)	(0.31188)	(0.22392)	(0.08550)
	[0.23884]	[-0.42830]	[-1.02930]	[0.32511]
D(LRERATE(-1))	-0.286069	0.430832	-0.438424	-0.081994
	(0.43162)	(0.68215)	(0.48976)	(0.18700)
	[-0.66278]	[0.63158]	[-0.89518]	[-0.43847]
D(LRERATE(-2))	-0.290477	0.918091	-0.257167	-0.138823
	(0.35976)	(0.56858)	(0.40822)	(0.15587)
	[-0.80741]	[1.61470]	[-0.62996]	[-0.89063]
D(LRERATE(-3))	-0.140685	0.762706	-0.044545	-0.042564
	(0.32978)	(0.52119)	(0.37420)	(0.14288)
	[-0.42661]	[1.46339]	[-0.11904]	[-0.29790]
D(LRGDP_WORLD(-1))	0.339356	0.720224	-0.683251	0.168730
	(0.83629)	(1.32171)	(0.94895)	(0.36233)
	[0.40579]	[0.54492]	[-0.72001]	[0.46568]
D(LRGDP_WORLD(-2))	-0.395642	-0.649402	0.112987	0.097089
	(1.02600)	(1.62152)	(1.16421)	(0.44452)
	[-0.38562]	[-0.40049]	[0.09705]	[0.21841]
D(LRGDP_WORLD(-3))	0.541982	1.158440	-1.462153	-0.015429
	(0.73460)	(1.16099)	(0.83356)	(0.31827)
	[0.73779]	[0.99781]	[-1.75412]	[-0.04848]
C	-0.003732	0.062688	0.039922	0.004034
	(0.02677)	(0.04231)	(0.03038)	(0.01160)
	[-0.13941]	[1.48148]	[1.31407]	[0.34778]

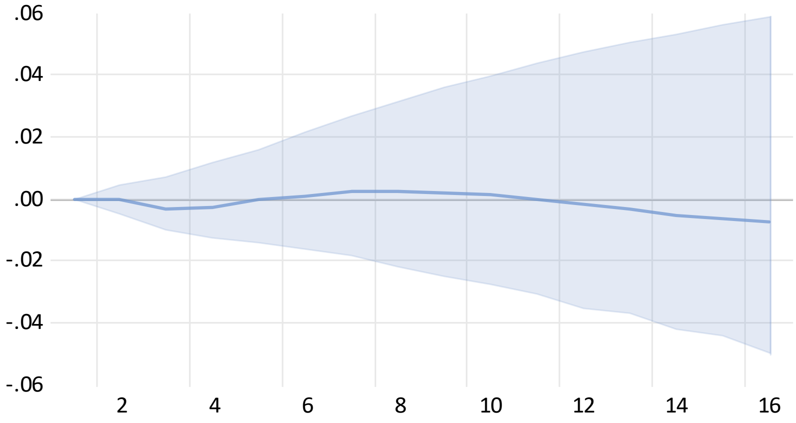
R-squared	0.397655	0.387019	0.575942	0.617852
Adj. R-squared	-0.581156	-0.609076	-0.113151	-0.003138
Sum sq. resids	0.001833	0.004578	0.002360	0.000344
S.E. equation	0.015136	0.023921	0.017175	0.006558
F-statistic	0.406263	0.388536	0.835797	0.994947
Log likelihood	72.10614	62.03674	69.32592	90.50741
Akaike AIC	-5.282377	-4.366976	-5.029629	-6.955219
Schwarz SC	-4.588077	-3.672676	-4.335330	-6.260919
Mean dependent	-0.000383	0.045504	0.002075	0.009430
S.D. dependent	0.012037	0.018858	0.016278	0.006547

Determinant resid covariance (dof adj.)	2.61E-17
Determinant resid covariance	4.56E-19
Log likelihood	339.6805
Akaike information criterion	-25.42550
Schwarz criterion	-22.44993
Number of coefficients	60

Figure 3 shows impulse responses with bootstrapped 95% confidence intervals based on the model presented in Table 5. The impulse variable is real exchange rate, while the response variable is the real trade balance. The impulse responses are cumulated over 16 quarters.

Figure 3: Cumulative impulse responses for the data including real exchange rates

Accumulated Response of TBALANCE to LRERATE Cholesky One S.D. (d.f. adjusted) Innovation
95% CI using Standard percentile bootstrap with 999 bootstrap repetitions



The impulse response does not appear to be statistically significantly different from zero. It fluctuates around zero with a period of approximately one year. Unlike the Ukrainian case, no convergence to 0.7 is observed.

Next, we estimate the VECM using the dataset that includes the real effective exchange rate. As suggested by the majority of information criteria, we use only one lag.

Table 6: VECM for data including real effective exchange rate

Vector Error Correction Estimates

Sample (adjusted): 2019Q1 2024Q4

Included observations: 24 after adjustments

Standard errors in ( ) & t-statistics in [ ]

Lags interval (in first differences): 1 to 1

Endogenous variables: TBALANCE LRGDP_SA LREER_BROAD LRGDP_WORLD

Deterministic assumptions: Case 3 (Johansen-Hendry-Juselius): Cointegrating relationship includes a constant. Short-run dynamics include a constant.

Cointegrating Eq:	CointEq1
TBALANCE(-1)	1.000000
LRGDP_SA(-1)	0.073695
	(0.02381)
	[3.09490]

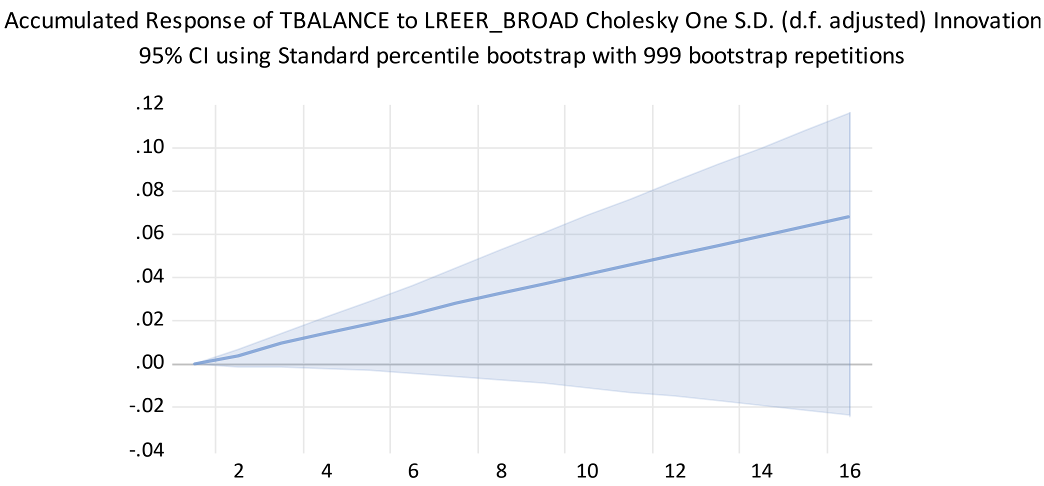
LREER_BROAD(-1)	-0.474160			
	(0.11121)			
	[-4.26350]			
LRGDP_WORLD(-1)	-0.131395			
	(0.13352)			
	[-0.98405]			
C	1.468757			

Error Correction:	D(TBALANCE)	D(LRGDP_SA)	D(LREER_BROAD)	D(LRGDP_WORLD)
COINTEQ1	-0.939052	-1.379061	0.188261	0.016708
	(0.25957)	(0.55437)	(0.57661)	(0.16415)
	[-3.61776]	[-2.48764]	[0.32649]	[0.10178]
D(TBALANCE(-1))	0.028703	0.765220	-0.229520	0.116731
	(0.19609)	(0.41880)	(0.43560)	(0.12401)
	[0.14637]	[1.82719]	[-0.52690]	[0.94131]
D(LRGDP_SA(-1))	-0.091008	-0.094282	0.074974	-0.054466
	(0.09400)	(0.20077)	(0.20883)	(0.05945)
	[-0.96812]	[-0.46961]	[0.35903]	[-0.91617]
D(LREER_BROAD(-1))	-0.146583	-0.552640	-0.102574	0.013674
	(0.12123)	(0.25891)	(0.26930)	(0.07667)
	[-1.20914]	[-2.13447]	[-0.38089]	[0.17836]
D(LRGDP_WORLD(-1))	0.605980	0.009498	0.886594	0.450922

	(0.33434)	(0.71406)	(0.74272)	(0.21144)
	[1.81246]	[0.01330]	[1.19371]	[2.13261]
C	-0.000352	0.055037	-0.006339	0.007636
	(0.00581)	(0.01241)	(0.01291)	(0.00367)
	[-0.06063]	[4.43468]	[-0.49104]	[2.07793]
R-squared	0.476502	0.310345	0.146711	0.292211
Adj. R-squared	0.331086	0.118774	-0.090313	0.095604
Sum sq. resids	0.001649	0.007523	0.008139	0.000660
S.E. equation	0.009572	0.020443	0.021264	0.006053
F-statistic	3.276821	1.620000	0.618970	1.486265
Log likelihood	80.97147	62.76006	61.81573	91.96869
Akaike AIC	-6.247623	-4.730005	-4.651310	-7.164057
Schwarz SC	-5.953109	-4.435491	-4.356797	-6.869544
Mean dependent	0.000235	0.049019	0.005179	0.009218
S.D. dependent	0.011704	0.021777	0.020364	0.006365
Determinant resid covariance (dof adj.)		3.69E-16		
Determinant resid covariance		1.17E-16		
Log likelihood		304.0281		
Akaike information criterion		-23.00234		
Schwarz criterion		-21.62794		
Number of coefficients		28		

Based on the model presented in Table 6 we compute cumulative impulse responses for the real trade balance.

Figure 4: Cumulative impulse responses for the data including real effective exchange rates



As Figure 4 shows, the impulse response does not converge to a constant value. Instead, it diverges toward infinity. In any case, the impulse response is statistically indistinguishable from zero.

5. Conclusion

I attempted to replicate the Ukrainian model of the equilibrium effective exchange rate using quarterly data from Uzbekistan. Two alternative vector error correction models were estimated for the period between 2018Q3 and 2024Q4. One model uses the real exchange rate, and the other uses the real effective exchange rate. Neither model replicated the Ukrainian pattern. In both cases, there was no convergence to a constant value. Furthermore, the impulse responses were not statistically significantly different from zero.

6. References

Weber, E., & Kirchner, R. (2009). Methodological note to presentation “Equilibrium exchange rate in Ukraine” (Technical Note TN/01/2009). German Advisory Group (German Economic Team)

Appendix

This is the EViews code used for the estimations.

```
'Construction of a model of the equilibrium exchange rate for Uzbekistan

'******************************************

'' Initial settings

'******************************************

 ' Specify path where the data are located

cd "c:\kkholodilin\Projekty\GET\Data\"

%sInFile = "Data_Equilibrium_exchange_rate_Uzbekistan.xlsx" ' Name of input file

%sOutFile1 =  "Fig_Uzbekistan_original_data_Equilibrium_exchange_rate.jpg" ' Name of
output figure file

%sOutFile2=  "Fig_Uzbekistan_transformed_data_Equilibrium_exchange_rate.jpg" ' Name
of output figure file

%sOutFile2 = "Fig_Equilibrium_exchange_rate_IRF.jpg"

wfcreate(wf="Data", page="quarterly") q 2016 2025 ' Create working file

' ********************************

'        Import data

'*********************************

import %sInFile range=data colhead=1 namepos=last na="#N/A" @freq Q 2016Q1 @smpl
@all

'Plot graph of original series
```

```
graph gr_variables.line(m) RGDP GDP_deflator RExports RImports (rexports-rimports)
RGDP_world ERate_inoff REER_broad REER_narrow  ' Create graph with multiple lines and
specify variables to be plotted

gr_variables.addtext(t) "Variables of original the model" ' Add overall title to graph

gr_variables.axis(left) -label grid zeroline ' Add grid lines

gr_variables.save(options) {%sOutFile1} ' Save graph


' *********************************

'       Transform data

'*********************************


smpl 2018q3 2024q4


'--- Compute real exchange rate


series ERate_index = 100 * (ERate_inoff / ERate_inoff(1))

series Deflator_index = 100 * (GDP_deflator / GDP_deflator(1))

series RERate = ERate_index / Deflator_index


'--- Take logs


series lrgdp = log(rgdp)

series lrgdp_world = log(rgdp_world)

series lrexports = log(rexports)

series lrimports = log(rimports)

series lerate = log(ERate_inoff)

series lrerate = log(RERate)
```

```
series lreer_broad = log(REER_broad)

series lreer_narrow = log(REER_narrow)


'--- Perform seasonal adjustment


lrgdp.x13(mode=add)

rename lrgdp_d11 lrgdp_sa


lrexports.x13(mode=add)

rename lrexports_d11 lrexports_sa


lrimports.x13(mode=add)

rename lrimports_d11 lrimports_sa


series tbalance = log(lrexports_sa / lrimports_sa)


'--- Plot graph of transformed series

graph gr_variables_transf.line(m) LRGDP_sa GDP_deflator LRExports_sa lRImports_sa (LRExports_sa - lRImports_sa) LRGDP_world LRERate LREER_broad LREER_narrow ' Create graph with multiple lines and specify variables to be plotted

gr_variables_transf.addtext(t) "Transformed variables of the model" ' Add overall title to graph

gr_variables_transf.axis(left) -label grid zeroline ' Add grid lines

'gr_variables.save(options) {%sOutFile2} ' Save graph


' ********************************************

'       Perform Johansen cointegration test
```

```
'*********************************************

' Create a group of variables (replace with your variable names)
group vars_rerate tbalance lRGDP_sa LRERate lRGDP_world
group vars_reer tbalance lRGDP_sa LREER_broad lRGDP_world

coint(method=s) vars_rerate
coint(method=s) vars_reer

' ********************************
'       Estimate VAR and VECM
'*********************************

'---- Real exchange rate

var var_rerate.ls 1 1 vars_rerate '@ @trend
show var_rerate.laglen(4, vname=v1)
var vecm_rerate.ec 1 3 vars_rerate

var_rerate.impulse(20, a) tbalance @ lrerate
vecm_rerate.impulse(16, se=boot, a) tbalance @ lrerate

'---- Real effective exchange rate
var var_reer.ls 1 2 vars_reer
show var_reer.laglen(4, vname=v2)
var vecm_reer.ec 1 1 vars_reer
```

```
'var_reer.impulse(16, a) tbalance @ lreer_broad

vecm_reer.impulse(16, se=boot, a) tbalance @ lreer_broad
```